Performance Assessment Task

Boxes
Grade 6

The task challenges a student to demonstrate understanding of equality. A student must make sense of different mathematical contexts for equality such as scales and balance beams. Using a pictorial representation of the weight of different numbered blocks on either side of a scale, a student must use mathematical reasoning to determine which box is the heaviest. Given additional scenarios with a balance scale, a student must continue to make sense of equivalent weights to determine in each scenario the heaviest box.

Common Core State Standards Math - Content Standards

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

Reason about and solve one-variable equations and inequalities.
6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

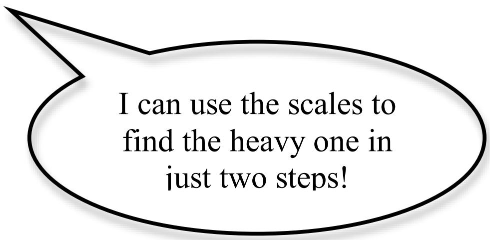
Grade Level	Year	Total Points	Core Points	% At Standard
6	2009	10	5	56 %

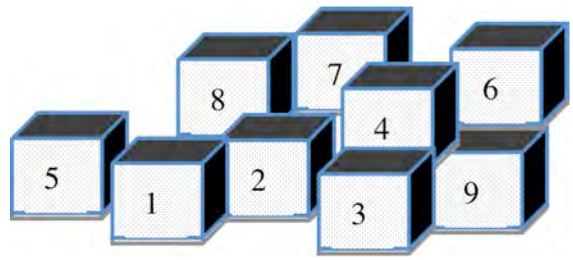
Boxes

This problem gives you the chance to:
• show understanding of a mathematical situation
• explain your reasoning

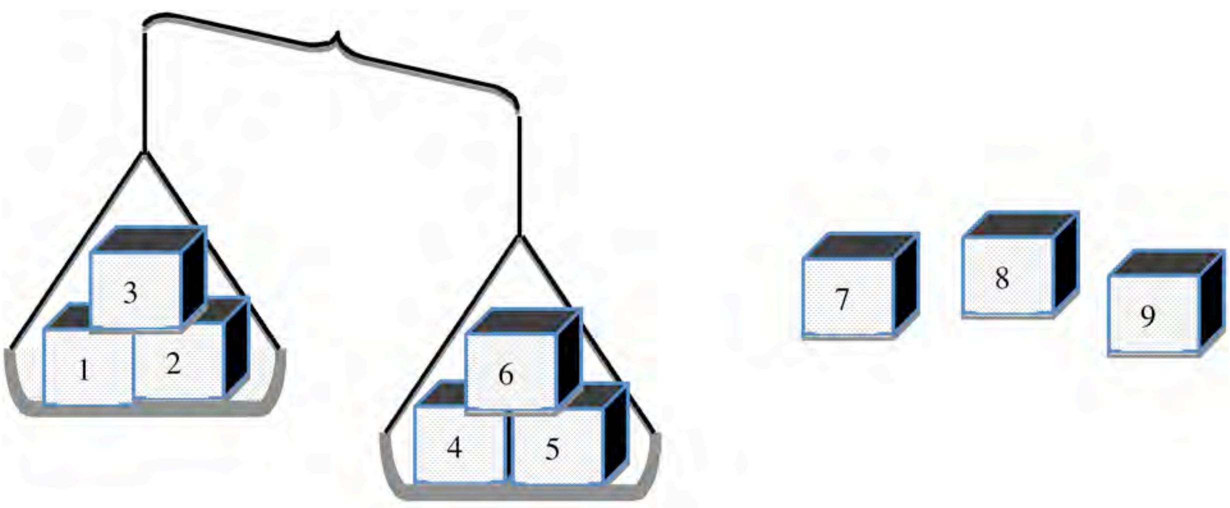
There are nine small boxes.
They all look exactly the same but one is a bit heavier than the others.

Jake says,





This is what Jake does first.



1. Explain what Jake now knows about the heavy box.

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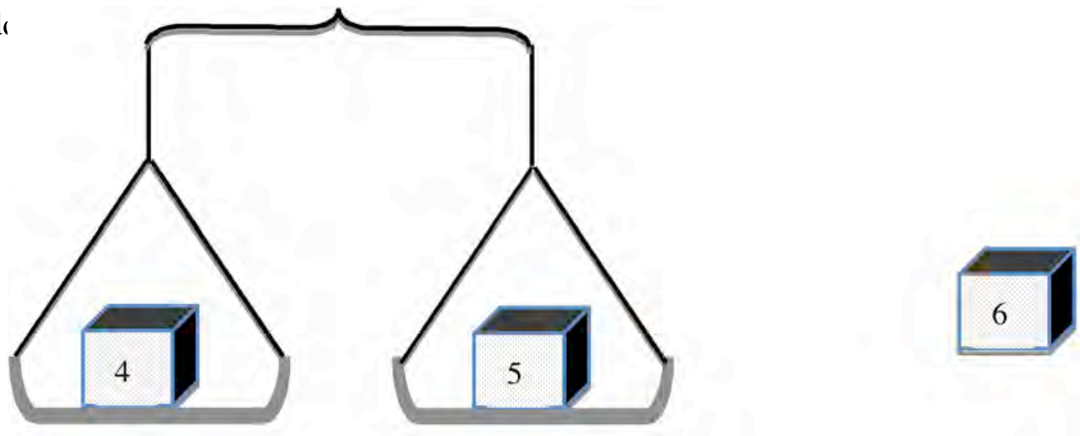
Then Jake d



2. Which is the heavy box? ______________________________

3. Explain how you know.

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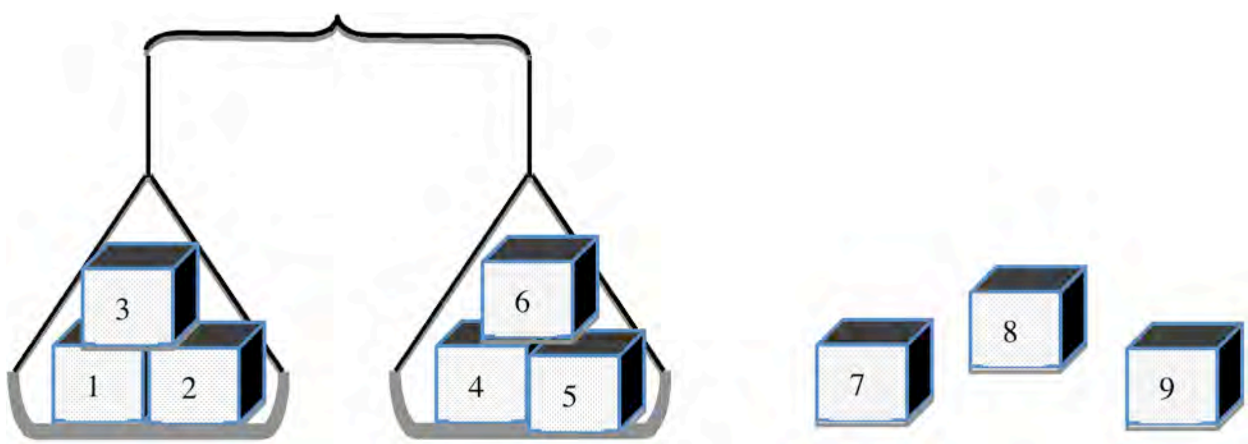
4. Suppose the scales showed this the first time instead.



What should Jake do now to find the heavy box?

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Boxes Rubric		
The core elements of performance required by this task are: • show understanding of a mathematical situation • explain your reasoning Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: It is **4, 5, or 6** Accept RHS or lower scale pan.	2	2
2. Gives correct answer: **6**	1	1
3. Gives correct answer: **4 and 5 weigh the same so 6 is the heavy one.** *Partial credit* 4 and 5 are the same weight.	2 (1)	2
4. Gives correct answer such as: Weigh 7 and 8 and leave 9 out. If 7 and 8 weigh the same, 9 is the heavy one. If the scales do not balance, the heavy parcel is in the lower scale pan. Incomplete explanation. Weighs 7 and 8 and nothing else.	3 (2) (1)	3
Total Points		**8**

Boxes

Work the task and look at the rubric. What are some of the big mathematical ideas a student needs to understand to be successful on this task? ________________________

Look at part one, what do you know from looking at the scale. How many of your students:

- Knew that the heaviest box was 4, 5, or 6?___________
- Thought the higher the number the heavier the box?________

What were students misunderstanding about the scale? How often do students have opportunities to work with balance scales? What experiences have they had this year?

Now look at part 2, looking at boxes 4, 5, and 6. How many of your students thought:

- Box 6 was the heaviest box and could give a reason for that?__________
- Box 5 was the heaviest box because its slightly lower (looking at the picture)?_____
- Box 5 is heavier because it's a larger number?________
- Neither 4 or 5, they're the same weight (didn't consider box 6)?______

What was the important logic that students should discern from the diagram?

Now look at student work in part 3, what should Jake do next. What should students know so far? How many of your students:

- Could use the constraint of weighing only twice and write a complete explanation of what to do next that includes all the outcomes?____________
- Only thought to weigh 7 and 8 (any two of the remaining boxes), but didn't talk about how that eliminated the need for any further weighing?_________
- Said to see which of 7, 8, or 9 is the heaviest, but don't give any details?_______
- Mention that either 7, 8, or 9 is the heaviest, but give no directions?_________
- Suggested weighing the boxes 1 by 1?__________
- Weigh 7, 8, and 9 against one of the other sets of 3?_______
- Talked about adding numbers?_______

What opportunities do students have to develop experiments for themselves to test out ideas?
What activities or experiences help students develop their logical reasoning skills?
How would you follow up on this task in your classroom?

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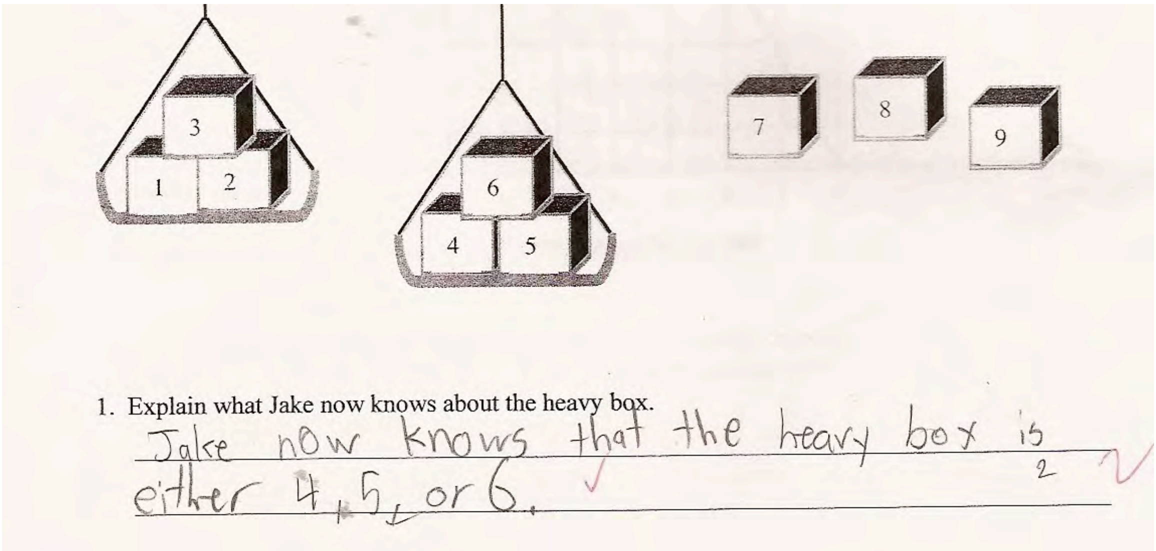
Looking at Student Work on Boxes

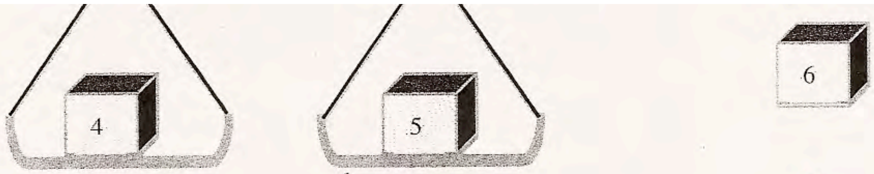
Student A is able to analyze and interpret the meanings of the scales in part 1 and 2. The student describes what is known from the drawing in part 4 and then gives a complete explanation of what do next and how to interpret the results of that next step.

Student A



1. Explain what Jake now knows about the heavy box.

Jake now knows that the heavy box is either 4, 5, or 6.

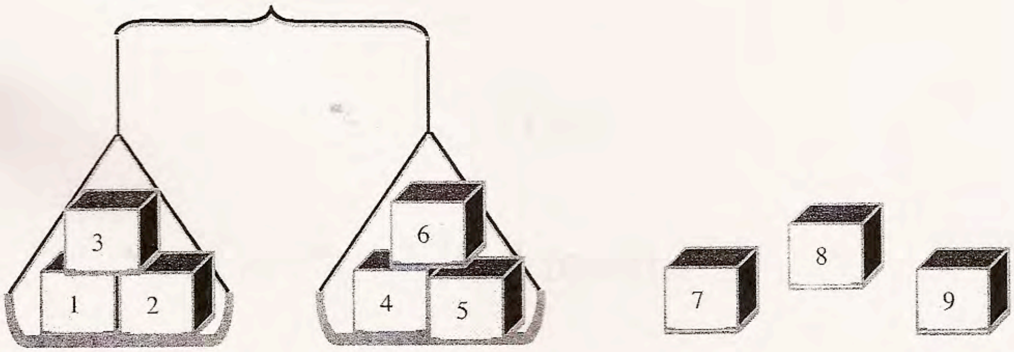


2. Which is the heavy box? 6 ✓ 1

3. Explain how you know. 2

I know this because there is only 1 heavy box and 4 and 5 are equal so 6 has to be the heavy box that tipped the scale in step 1.

4. Suppose the scales showed this the first time instead.



What should Jake do now to find the heavy box?

Jake now knows that 1, 2, 3, 4, 5, and 6 ~~aren't~~ don't contain the heavy box so he should weigh 7, and 8 and if 7 or 8 tips the scale that one is the heavy one, but if they're equal than 9 is the heavy box (see above) (2

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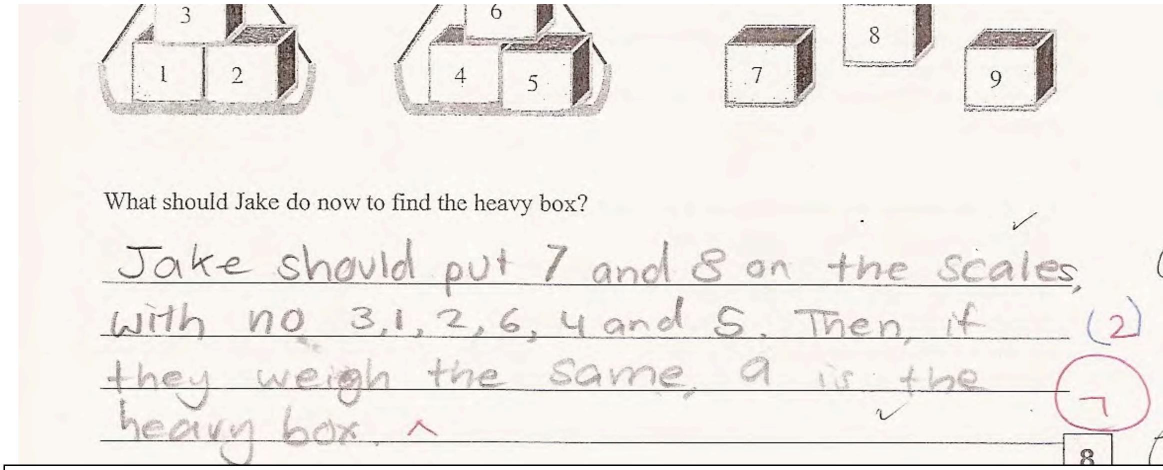
While many students could think about the specific situations in parts 1, 2, and 3, they couldn't construct the reasoning chain of what to do next in part 4. Student B is able to eliminate several boxes and give a logical next step. However, Student B talks about only 1 of the two possible outcomes.

Student B



3 1 2 6 4 5 7 8 9

What should Jake do now to find the heavy box?

Jake should put 7 and 8 on the scales with no 3, 1, 2, 6, 4 and 5. Then, if they weigh the same, 9 is the heavy box.

(2)

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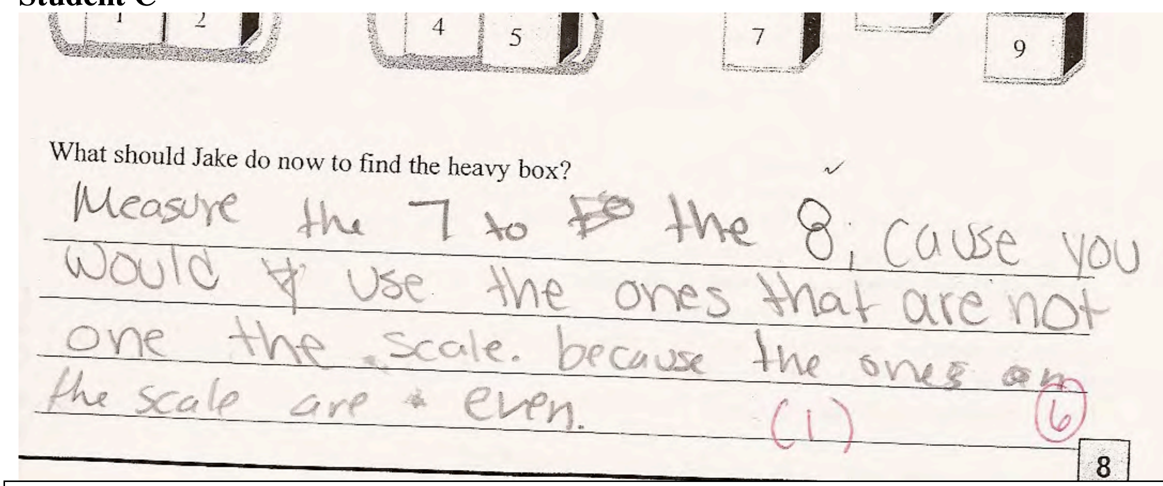
Student C offers a next step, explaining why it comes after the one shown. The student does not explain how this will help.

Student C



1 2 4 5 7 9

What should Jake do now to find the heavy box?

Measure the 7 to ~~F~~ the 8, cause you would use the ones that are not one the scale. because the ones on the scale are even.

(1)

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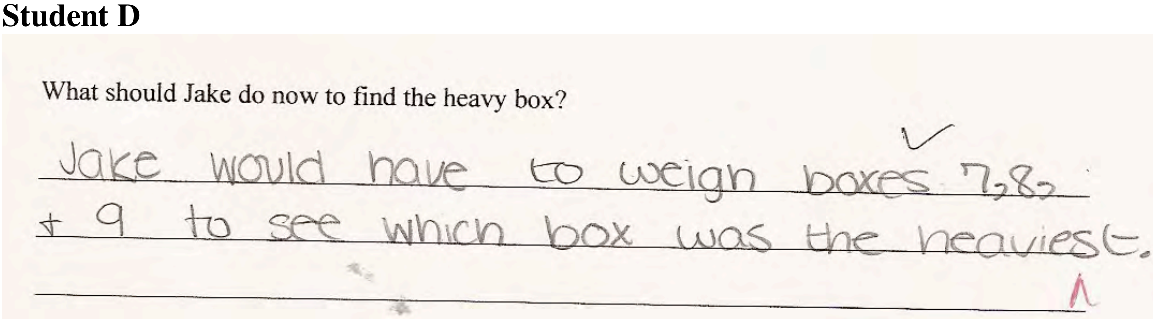
Student D also offers all possible next steps without explanation for how to interpret the results and solve the task of finding the heaviest box in that one final step.

Student D

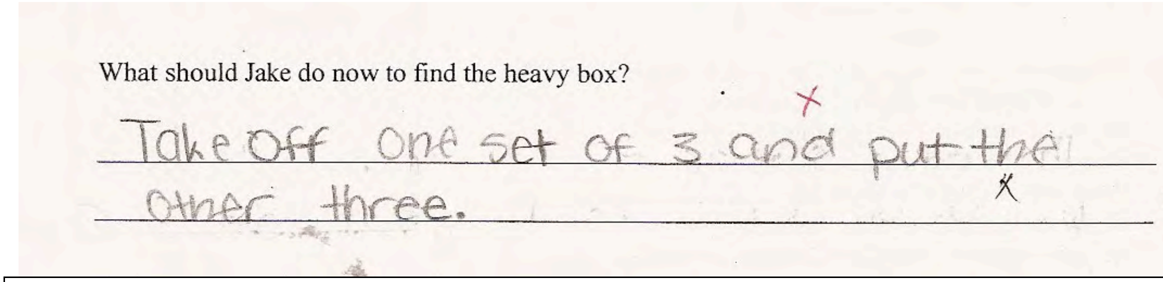


What should Jake do now to find the heavy box?

Jake would have to weigh boxes 7, 8, + 9 to see which box was the heaviest.

Student E has determined which three boxes might be heaviest, but does not give a specific next step.

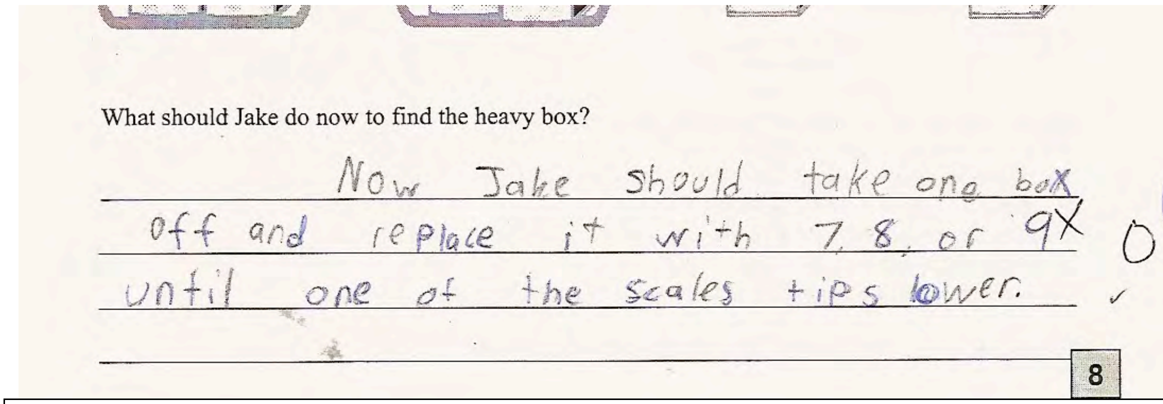
Student E


What should Jake do now to find the heavy box?

Take off one set of 3 and put the other three.

Student F gives a series of steps, rather than using the constraint of only two total steps to find the heaviest box.

Student F


What should Jake do now to find the heavy box?

Now Jake should take one box off and replace it with 7, 8, or 9 until one of the scales tips lower.

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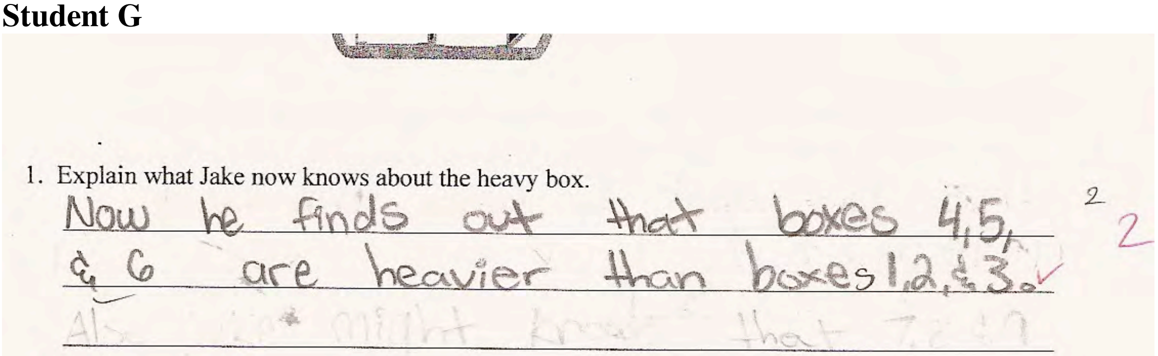
Student G is able to interpret the diagram in part 1 and follow the logic. The student can interpret the diagram in part 2, but doesn't make the logical inference about box 6 being the heaviest. The student doesn't see the significance of the second step. This is shown in the work in part 4 where the student lists all possible combinations of weighing the boxes against each other. *How do we help students develop their able to make inferences?*

Student G


1. Explain what Jake now knows about the heavy box.

Now he finds out that boxes 4,5, & 6 are heavier than boxes 1,2,&3.

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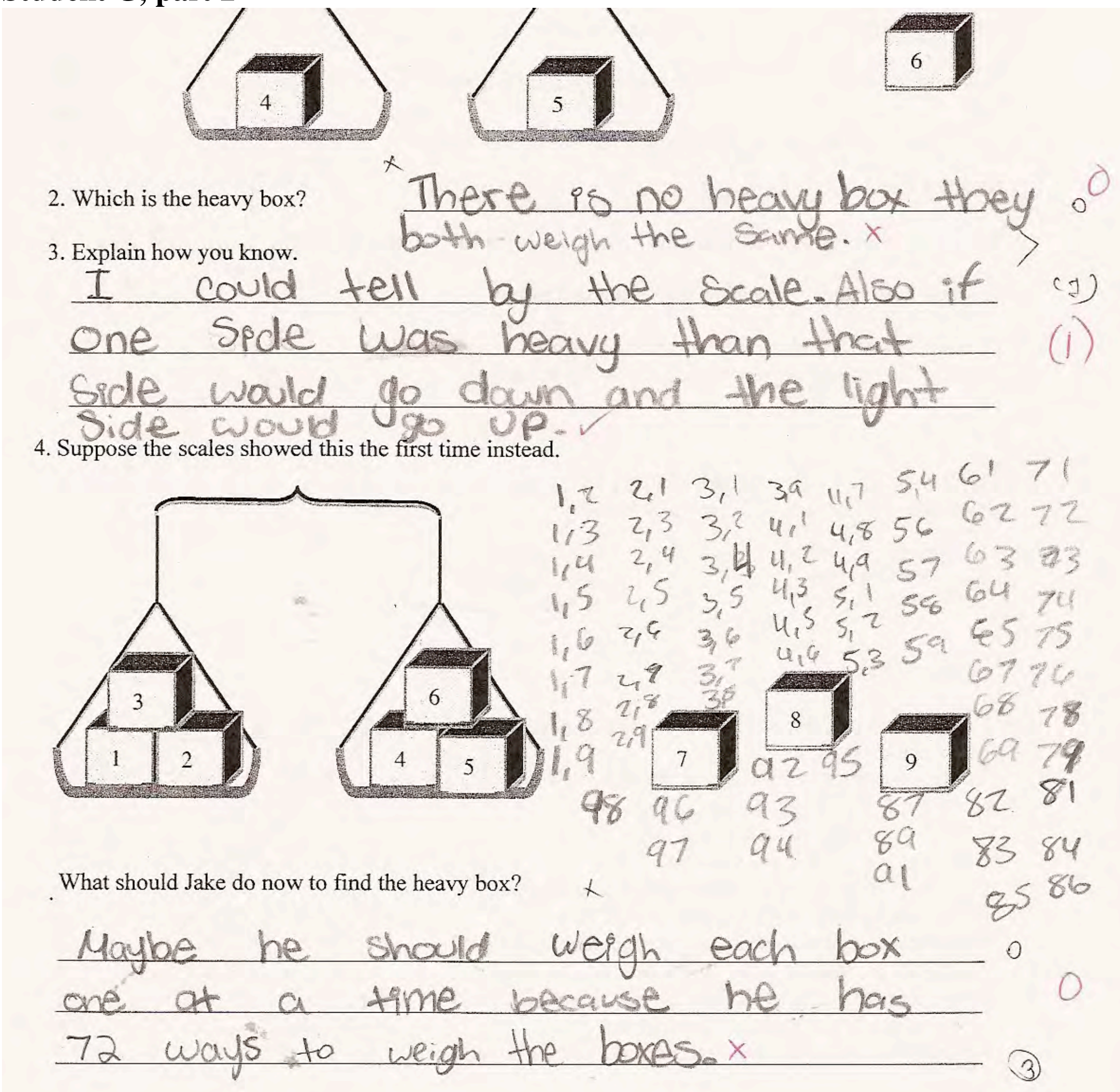
Student G, part 2



2. Which is the heavy box? There is no heavy box they both weigh the same.

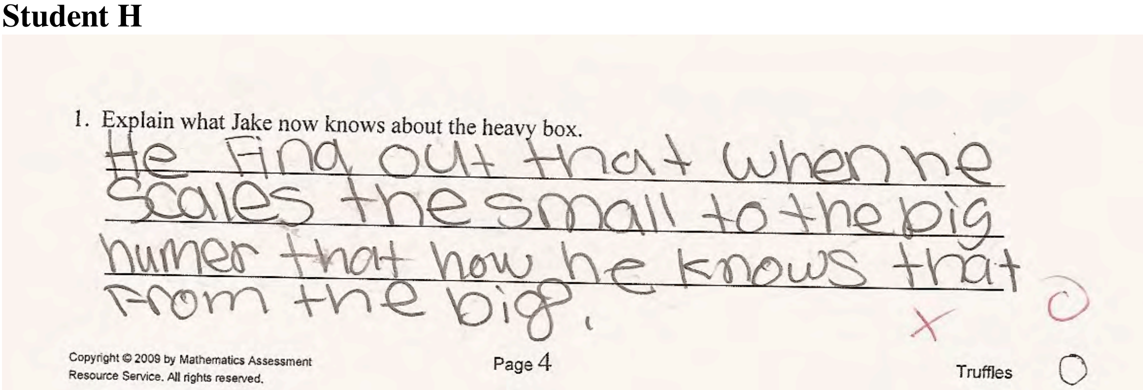
3. Explain how you know.
I could tell by the scale. Also if one side was heavy than that side would go down and the light side would go up.

4. Suppose the scales showed this the first time instead.

What should Jake do now to find the heavy box?
Maybe he should weigh each box one at a time because he has 72 ways to weigh the boxes.

Student H struggles with the meaning of the diagrams. In part 2 the student does not use language about measuring weight, but talks about length instead. The responses do not show an understanding of the math or context.

Student H



1. Explain what Jake now knows about the heavy box.
He find out that when he scales the small to the big numer that how he knows that from the big.

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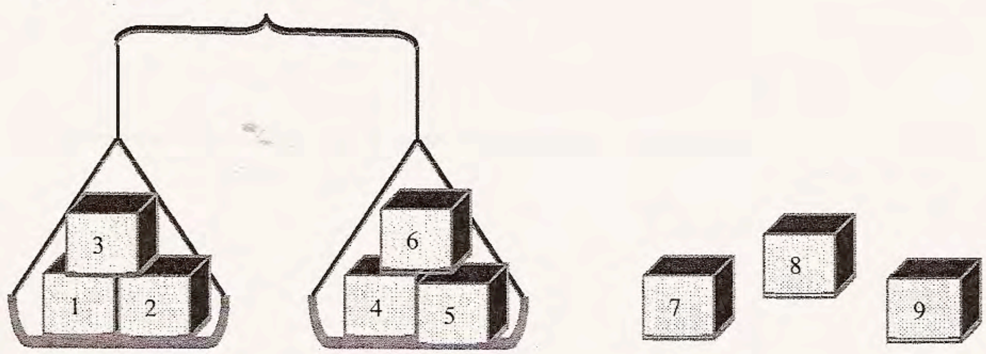
Student H, part 2

2. Which is the heavy box? none

3. Explain how you know.
I know because they are in the same length.

4. Suppose the scales showed this the first time instead.



What should Jake do now to find the heavy box?
He sould get more boxes and see what he will get next.

Students I and J have the misconception that the box numbers are weight.

Student I

1. Explain what Jake now knows about the heavy box.
The higher number the heavier.

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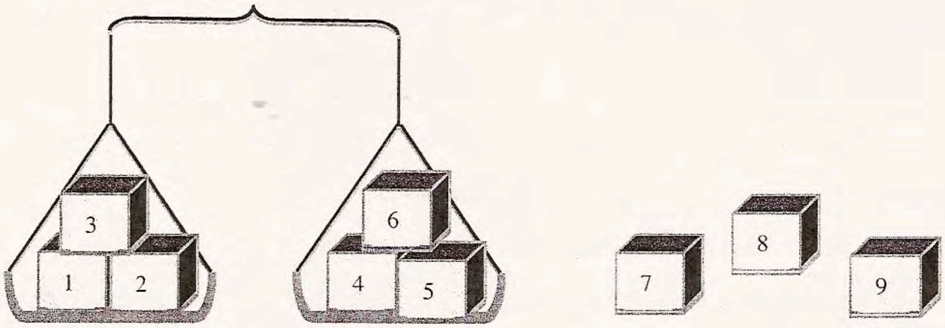
Student I, part 2

2. Which is the heavy box? 5

3. Explain how you know.

5 is a higher number than 4 which means that highest number is the heaviest.

4. Suppose the scales showed this the first time instead.



What should Jake do now to find the heavy box?

put another block on one scale and see what happens beacause there is thee same amount of blocks on the scales.

Student J

1. Explain what Jake now knows about the heavy box.

Because it matters what number is inside. Since 6 is the biggest number there in the scale he knows it would be heavy.

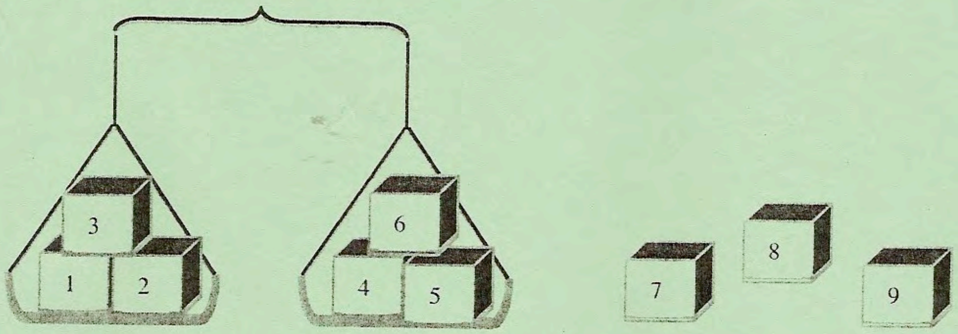
Student J, part 2

2. Which is the heavy box? 5 X 0

3. Explain how you know.

Because 5 is bigger than 4 so 5 must have a little more inside the box X 0

4. Suppose the scales showed this the first time instead.



What should Jake do now to find the heavy box? 0

1) Add them then which has the bigger number will be more heavy.

2) Another way is that you know the other one has bigger numbers so that will be heavy.

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6^{th} Grade Task 3 Boxes

Student Task	Show understanding of a mathematical situation and explain your reasoning.
Core Idea 3 Algebra and Functions	**Understand relations and functions, analyze mathematical situations, and use models to solve problems.** • Employ forms of mathematical reasoning and justification appropriately to the solution of a problem.

Mathematics of the task:
- Use diagrams of scales to think about weight and make inferences about heaviest
- Understand the constraint of only weighing twice
- Develop a logical train of thought about all possible results from weighing two items on a scale

Based on teacher observation, this is what sixth graders know and are able to do:
- Analyze the diagram in part 1 and interpret it in the context of the problem
- Analyze the diagram in part 2

Areas of difficulty for sixth graders:
- Understanding that the numbers on the boxes were names not weights
- Inferring that 6 is the heaviest when 5 and 4 weigh the same
- Determining the next step in part 4 and explaining how to interpret the outcomes

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MARS Test Task 3 Frequency Distribution and Bar Graph, Grade 6

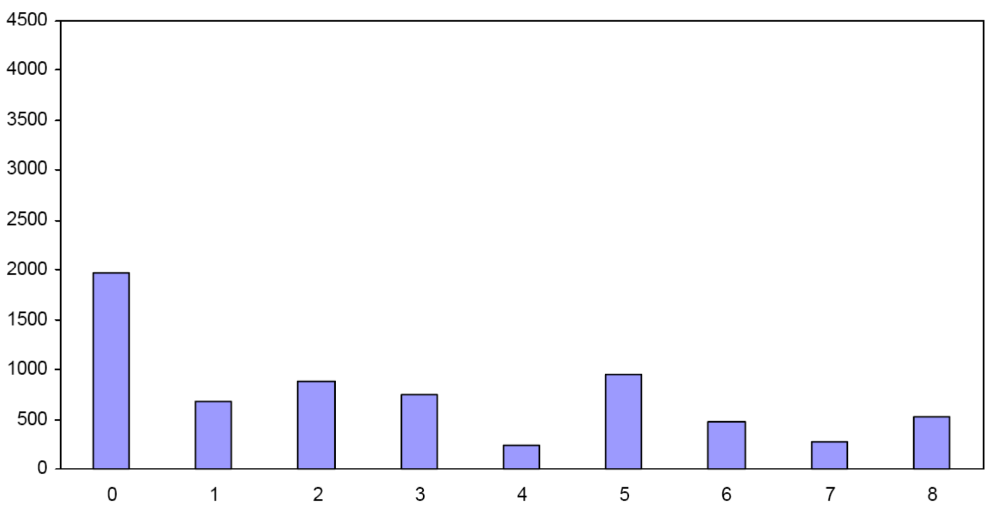
Task 3 - Boxes

Mean: 2.86 StdDev: 2.64

Table 32: Frequency Distribution of MARS Test Task 3, Grade 6

Task 3 Scores	Student Count	% at or below	% at or above
0	1969	29.3%	100.0%
1	676	39.4%	70.7%
2	879	52.5%	60.6%
3	743	63.6%	47.5%
4	243	67.2%	36.4%
5	945	81.3%	32.8%
6	470	88.3%	18.7%
7	264	92.2%	11.7%
8	523	100.0%	7.8%

Figure 41: Bar Graph of MARS Test Task 3 Raw Scores, Grade 6



The maximum score available for this task is 8 points.
The minimum score needed for a level 3 response, meeting standard, is 4 points.

Many students, 60%, could read the scale in part 1 and explain the results in the context of the problem. Some students, about 36%, could interpret the diagram in 1 and 2, and give some explanation in part 3. 33% of the students could give a full explanation in part 3 including making the inference that box 6 was the heaviest. 8% of the students could meet all the demands of the task including describing the next step in part 4 and explaining how to interpret all of the possible outcomes. 29% of the students scored no points on this task. 95% of the students with this score attempted the task.

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Boxes

Points	Understandings	Misunderstandings
0	95% of the students with this score attempted the task.	Students could not interpret the diagram in part 1. 21% of the students said that the larger the numbers the heavier the boxes.
2	Students could interpret the scale in part 1 and explain how the information helps identify the heaviest box.	Students had difficulty with interpreting the scale in part 2. 24% of the students thought box 5 was heaviest. 15% thought neither was heaviest. 7% thought box 4 was heaviest.
4	Students could interpret the scales in one and two and give some explanation in part 3.	Students had difficulty explaining part 3. 15% thought 5 was heavier because it's a bigger number. 7% that 5 was tilted slightly. 7% thought 4 was tilted. 20% said the two boxes weighed the same.
5	Students could interpret the scales in one and two and explain its significance in part 3.	Students struggled with developing the logic of the next step in part 4. 14% suggested weighing the boxes 1 by 1. 11% said to find out which of the boxes, 7,8, or 9 was heavier, but gave no directions. 9% said simply to weigh boxes 7 and 8. 8% said to put 7,8, and 9 on the pan with no explanation. 7% said to add all the numbers.
8	Students could interpret the scales in one and two and explain its significance in part 3. Students could also describe the next step in part 4 and give information to interpret the results from all the possible outcomes.	

Implications for Instruction

Students need to be able to look at balance scales and determine if the sides are equal or not equal. Students should be able to make simple deductions; such as if both sides are equal the heavy box is one of the boxes not on the scale. Many students did not seem to have experience with balance scales.

Students also need practice with logic problems, trying to figure out what they know and what they need to find. Many students did not understand how to do something in the fewest number of steps. For example, they may have suggested replacing 7,8, and 9 with 1,2, and 3 in part 4. This will tip the scales, but won't yield new information. If the scale is balanced now, then it is already known that 7, 8, or 9 is the heaviest box. Students don't often get the opportunity to use logic to solve problems.

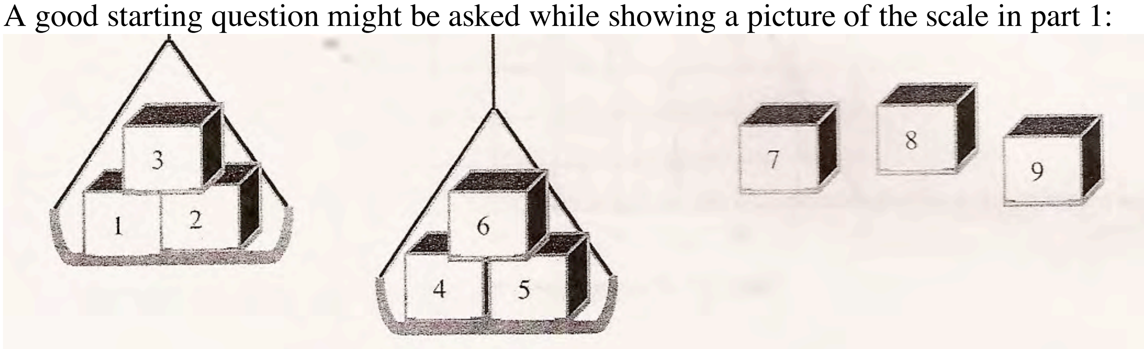
Ideas for Action Research

Re-engagement – Confronting misconceptions, providing feedback on thinking, going deeper into the mathematics. *(See overview at beginning of toolkit).*

1. Start with a simple problem to bring all the students along. This allows students to clarify and articulate the mathematical ideas.
2. Make sense of another person's strategy. Try on a strategy. Compare strategies.
3. Have students analyze misconceptions and discuss why they don't make sense. In the process students can let go of misconceptions and clarify their thinking about the big ideas.
4. Find out how a strategy could be modified to get the right answer. Find the seeds of mathematical thinking in student work.

This is a good task for planning a class discussion because there was so much confusion about the interpretation of the task and students not noticing information that would have disproved their interpretation.

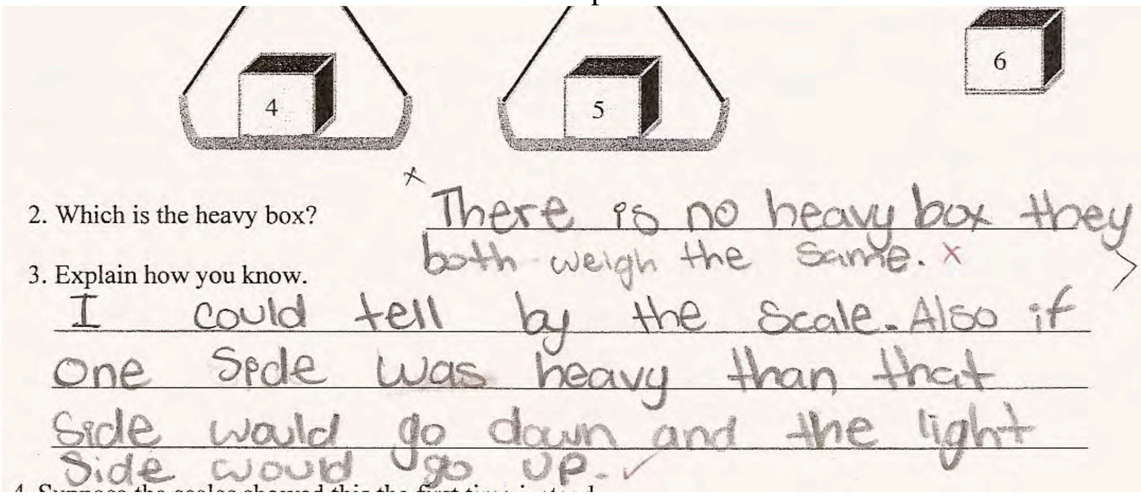
A good starting question might be asked while showing a picture of the scale in part 1:



"*Carolyn says that the box with the highest number is the heaviest. What information in the diagram might lead Carolyn to that conclusion?*" The idea is for students to tease out information that they know and don't know, to make ideas more explicit. After students find the logic in the initial statement, have them go beyond by examining more evidence.

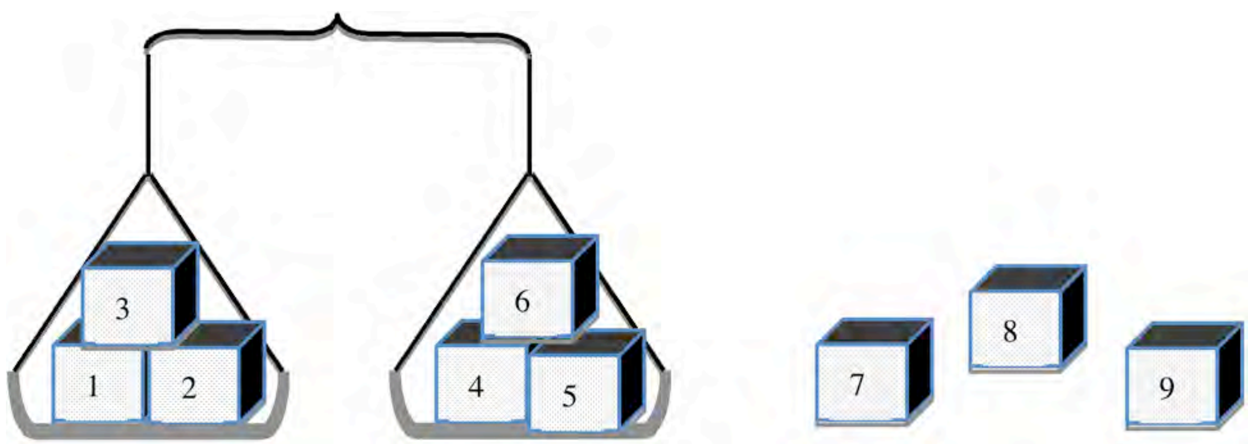
" *Is there any evidence that might convince Carolyn to change her mind?*"

Now have them look at the work of Adam on part 2.



"*What is Adam thinking? What evidence supports his idea?*" After some discussion, allowing students to really verbalize their thinking, then pose a follow up question, such as, "*Can you find any evidence to contradict Adam's thinking? Could you convince Adam to change his mind?*"

Finally explore some of the ideas presented in part 4. Have students examine this drawing:
This is what Jake does first.
Suppose the scales showed this the first time instead.



What should Jake do now to find the heavy box?

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Now pick some student work to cause some disequilibrium. For example you might ask, "*Jackie says that Jake should replace box 1,2, and 3 with boxes 7, 8, and 9. What do you think will happen? Bruce says that this is a waste of time. Do you agree or disagree? Why?*"

Often students appear to understand something by answering one question correctly. But then by posing further questions, their confusion reappears. It is important to not stop with just one correct solution or indication of understanding. An effective teacher keeps probing to make sure that students cement their understanding and can clearly articulate their ideas. *What work from the toolkit might you use next to push students thinking and why?* Discuss this with colleagues and decide how you might finish the lesson. *What are the big understandings that you want students to develop during the discussion?*

Ideas for Action Research Part 2

Students need exposure to a variety of logic problems to develop their ability to tease out information and follow things through to an evidence-based conclusion. There are a number of good sources for problems. Below are a couple of favorites from Creative Problem Solving in School Mathematics by George Lenchner.

1. Suppose you know that there are ten black, ten navy blue, and ten green socks in your drawer. The room is dark and you cannot turn on a light. What is the least number of socks that you must take out of your drawer to be certain that you meet each of the following conditions?
    a. You have a pair of the same color.
    b. You have a pair of black socks.
    c. You have one pair of each color.
2. Each of Ina, Jill, Louis and Miguel has a different favorite color among red, blue, green, and orange. No person's name contains the same number of letters as her or his favorite color. Louis and the boy who likes blue live in different parts of town. Red is the favorite color of one of the girls. What is each person's favorite color?
3. Alice, Betty, and Carolyn went on vacation. One went to Africa, one went to Bali, and one went to China. Only one of the following four statements is true.
    a. Carolyn went to Africa.
    b. Alice did not go to Bali.
    c. Carolyn did not go to Bali.
    d. Alice did not go to China.

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