## TOPIC

Mathematical Connections, measurement, and Problem Solving

## KEY QUESTION

How do you determine the number of nests at a pelican colony, which can contain hundreds or even thousands of nests, based on aerial photographs and information about the size and shape of each site?

## LEARNING GOALS

Students will:

- Use visual data to estimate the number of nests in a pelican colony
- Consider how to use and exclude data
- Make decisions about whether or not a solution meets the needs of a client
- Communicate the solution clearly to the client


## GUIDING DOCUMENTS

This activity has the potential to address many mathematics and science standards. Please see pages 4-6 for a complete list of mathematics and science standards.

## RECOMMENDED SUPPLIES FOR ALL MODEL-ELICITING ACTIVITIES

It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.

- Uncooked rice, small beans, or beads
- Small dots of paper from a hole punch
- Various containers and trays of both regular and irregular shapes for spreading out the small items
- String or ribbon (for outlining irregular shapes)
- Scissors
- Digital cameras
- Rulers
- Tape measures
- Compasses
- Protractors
- Copies of the Birds of North America report on the American White Pelican (in case students want more information about the species)


## WHAT ARE MODEL-ELICITING ACTIVITIES (MEAs)?

Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students' solutions to the task reveal explicitly how they are thinking about the given situation.

## THE PELICAN COLONIES MEA CONSISTS OF FOUR COMPONENTS:

1) Newspaper article: Students individually read the newspaper article to become familiar with the context of the problem. This handout is on page 7 .
2) Readiness questions: Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 8 .
3) Problem statement: In teams of three or four, students work on the problem statement for $45-90$ minutes. This time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students' first thoughts, and can be longer if you expect a polished solution and well-written letter. The handouts are on pages 912.
4) Process of sharing solutions: Each team writes their solution in a letter or memo to the client. Then, each team presents their solution to the class. Whole class discussion is intermingled with these presentations to discuss the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client.
In totality, each MEA takes approximately 2-3 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 14.

## RECOMMENDED PROGRESSION OF THE PELICAN COLONIES MEA

While other implementation options are possible for MEAs, it is recommended that the MEA be implemented in a cooperative learning format. Numerous research studies have proven cooperative learning to be effective at improving student achievement, understanding, and problem solving skills. In this method students will complete work individually (Newspaper article and readiness questions; as well as initial thoughts on the problem statement) and then work together as a group. This is important because brainstorming works best when students have individual time to think before working as a group. Students can be graded on both their individual and group contributions. Social skills' discussion at the beginning of the MEA and reflection questions at the end of the MEA are also essential aspects of cooperative learning.
Social Skills (3-5 minutes)
Students must be taught how to communicate and work well in groups. Several social skills that are essential to group work are decision- making, asking questions, and communicating and listening. The teacher can show part of a YouTube video and discuss aspects of these skills before beginning the MEA.
(http://www.youtube.com/user/flowmathemat ics)

## Newspaper Article and Readiness Questions:

The purpose of the newspaper article and the readiness questions is to introduce the students to the context of the problem.
(10 minutes): Give the article and the questions to the students the day before for homework. Then, in the next class, discuss as a class the answers to the readiness questions before beginning to discuss the problem statement.

## Problem Statement:

You may want to read the problem statement to the students and then identify as a class: a) the client that the students are working for and b) the product that the students are being asked to produce. Once you have addressed the points above, allow the students to work on the problem statement. Let the students know that they will be sharing their solution to the rest of the class. Tell students you that you will randomly pick a group member to present for each group. Tell the students that they need to make sure that everyone understands their group's solution so they need to be sure to work together well. The group member who will present can be picked by assigning each group member a number.

Working on the Problem Statement (35-50 minutes): Place the students in teams of three or four. Students should begin to work by sharing their initial ideas for solving the problem. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own teams. You may want to have the students choose a name for their team to promote unity.

Teachers' role: As they work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution. Try to
answer their questions with questions so that the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations.

Presentations of Solutions (15-30 minutes): The teams present their solutions to the class. There are several options of how you do this. Doing this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each presentation typically takes $3-5$ minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams' presentations but also to a) try to understand the other teams' solutions and b) consider how well these other solutions meet the needs of the client. You may want to offer points to students that ask 'good' questions of the other teams, or you may want students to complete a reflection page (explanation - page 4, form - page 15) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams' memos/letters, presentation overheads, and any other work you would like to look over or assess.

## ASSESSMENT OF STUDENTS' WORK

You can decide if you wish to evaluate the students' work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

Performance Level Effectiveness: Does the solution meet the client's needs?

Requires redirection: The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

Requires major extensions or refinements: The product is a good start toward meeting the client's needs, but a lot more work is needed to respond to all of the issues.

Requires editing and revisions: The product is on a good track to be used. It still needs modifications, additions or refinements.

Useful for this specific data given, but not shareable and reusable OR Almost shareable and reusable but requires minor revisions: No changes will be needed to meet the immediate needs of the client for this set of data, but not generalized OR Small changes needed to meet the generalized needs of the client.

Share-able or re-usable: The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

## IMPLEMENTING AN MEA WITH STUDENTS FOR THE FIRST TIME

You may want to let students know the following about MEAs:

- MEAs are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.
- MEAs often have more than one solution or one way of thinking about the problem.
- Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute presentation, and that they may use overhead transparencies or other visuals during their presentation.
- Let the students know that you won't be answering questions such as "Is this the right way to do it?" or "Are we done yet?" You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.
- Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.
- If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.



## OBSERVING STUDENTS AS THEY WORK ON THE PELICAN COLONIES MEA

You may find the Observation Form (page 12) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out "real-time" as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

## PRESENTATION FORM (Optional)

As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 13. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the pelican colonies MEA after hearing of the other teams' solutions.

## STUDENT REFLECTION FORM

You may find the Student Reflection Form (page 15) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations.

## STANDARDS ADDRESSED

## nCTM Mathematics Standards

## Numbers and Operations:

- Understand and use ratios and proportions to represent quantitative relationships
- Develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use
- Judge the reasonableness of numerical computations and their results
Algebra
- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations
- Use symbolic algebra to represent and explain mathematical relationships
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships
- Draw reasonable conclusions about a situation being modeled
Geometry
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture
Measurement
- Solve simple problems involving rates and derived measurements for such attributes as density
- Analyze precision, accuracy, and approximate error in measurement situations
Data Analysis and Probability
- Find, use, and interpret measures of center and spread, including mean and inter quartile range
- Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatter plots
Problem Solving
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving
Reasoning and Proof
- Develop and evaluate mathematical arguments and proofs
Communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others


## Connections

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics
Representation
- Use representations to model and interpret physical, social, and mathematical phenomena


## NRC SCIENCE STANDARDS

Inquiry

- Use appropriate tools and techniques to gather, analyze and interpret data
- Develop descriptions, explanations, predictions, and models using evidence
- Think critically and logically to make the relationships between evidence and explanations
- Recognize and analyze alternative explanations and predictions
- Communicate scientific procedures and explanations
- Use mathematics in all aspects of scientific inquiry


## Common Core Math Standards

- 5.G Graph points on the coordinate plane to solve real-world and mathematical problems.
- 5 MD-2: represent and interpret data
- 6 G-1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- 6 SP-1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.
- 6 SP-2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- 6 SP-3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- 6 SP-4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
- 6 SP- 5 Summarize numerical data sets in relation to their context, such as by:
a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was
measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
- 7 SP Use random sampling to draw inferences about a population
- High School S-ID Summarize, represent, and interpret data on a single count or measurement variable.

2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

- High school S-IC-1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.


## Standards for Mathematical Practices integration with MEAs

| Mathematical Practice | How it occurs in MEAs |
| :--- | :--- |
| 1. Make sense of <br> problems and persevere <br> in solving them. | As participants work <br> through iterations of their <br> models they continue to <br> gain new insights into <br> ways to use mathematics <br> to develop their models. |
| The structure of MEAs |  |
| allows for participants to |  |
| stay engaged and to have |  |
| sustained problem |  |


|  | solving experiences. |
| :---: | :---: |
| 2. Reason abstractly and quantitatively | MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically. |
| 3. Construct viable arguments and critique the reasoning of others. | Throughout MEAs while groups are working and presenting their models. |
| 4. Model with mathematics. | This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision. |
| 5. Use appropriate tools strategically. | Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices. |
| 6. Attend to precision. | Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic. |
| 7. Look for and make use of structure. | Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions. |
| 8. Look for and express regularity in repeated reasoning. | As participants develop their models the patterns they notice can assist in their model development. |

## Pelican Colonies - Part A

Read the following article and complete the individual exercise that follows.
The American White Pelican (scientific name: Pelecanus erythrorhynchos) is a large waterbird that is found mainly in the northwestern part of North America during the breeding season, and the coasts of the southern United States and Mexico during the winter. This species nests on islands in large lakes or along rivers, or on high dry areas in inland marshes. The birds make their nests on flat or mildly sloped areas with little vegetation. They may nest at sites where the substrate is soil, gravel, rock, or sand. Their nests are simple shallow depressions in the ground (they do not "build" nests). Pelicans are colonial nesters, which means that large groups of breeding pairs make their nests in one area, and the nests are very close together.

After the female pelican lays her eggs (almost always 2 eggs), the parents take turns incubating the eggs. Eggs must be kept at the right temperature while the chicks are developing inside; if the parents do not sit on the eggs to keep them warm in cold weather and keep them shaded from the hot sun, the developing chick will not survive. When one parent is sitting on the nest, the other parent leaves the nest site and flies to the feeding grounds to catch fish.


Knopf \& Evans 2004

When pelicans are nesting, they are very shy and are easily disturbed. If humans enter a breeding colony, the adult birds will leave their nests. Pelicans may also be disturbed by loud motorboats or other noises. If the eggs are left unguarded in the nest, they may become too cold or too hot (if the sun is shining on them). Another danger to unguarded eggs are predators like gulls, coyotes, and foxes. With a significant disturbance, the adults may not return to the colony at all.

The American White Pelican population in the Americas was once considered threatened. Population numbers were low as a resuit or numan disturbance and hunting, loss of nesting habitat due to changing water levels, and possibly contaminants in the environment that lead to a decrease in the thickness of pelican egg shells. Since the 1960's, pelican populations have slowly been growing as a result of their breeding colonies being protected from humans and their foraging and breeding habitats being protected from flooding or drainage. Wildlife researchers and managers monitor pelican populations every year to determine where nesting colonies are located and to estimate the total population size within and across colony sites.

## Individual Exercise

1. Make a list of all the things you would need to take into consideration if you were developing a plan to estimate the number of nests at a pelican colony.
2. For each of the factors that you listed above, explain why you would need to take that into consideration.
3. Once you have finished with numbers $1 \& 2$ above, request the packet from the United States Fish and Wildlife Service. Read the included memo individually.

## Memorandum

To: Research Team
From: Alice Heart
Wildlife Biologist, U. S. Fish and Wildlife Service
Subject: American White Pelican monitoring and management
Welcome to the United States Fish and Wildlife Service's pelican monitoring and management group. Wildlife biologists at the U. S. Fish and Wildlife Service monitor breeding colonies of American White Pelicans each year in order to determine which areas birds are using as colony sites, and to estimate the population size of each colony. Knowing where the birds are nesting helps us decide which sites we might undertake conservation efforts at, and what those efforts should be. Nesting colonies in lakes that are used heavily by recreational boaters might be protected by placing restrictions on boating in nearby waters. Colonies that are near wooded areas might require fencing to protect adult birds, eggs, and chicks from mammalian predators.

If the safety and reproductive success of the birds were the only factors under consideration, it would be preferable to protect every site known to have breeding colonies. However, this is not a realistic option. There are many constraints on conservation actions to protect breeding colonies, including cost and political implications. Therefore, scientists can only protect a subset of known colonies. One key criterion used to determine which sites should be protected is the number of bird nests. Sites with the largest number of nests are considered more important to protect. By protecting the colonies with the largest numbers of nests, we can protect as large a portion of the population as possible.

The U.S. Fish and Wildlife Service needs a procedure to estimate the number of nests at each pelican colony. Because pelicans are very sensitive to disturbances while they are incubating their eggs, we are not able to physically walk through every colony and count nests (this would also take too much time and cost too much!). We have hired pilots to fly our biologists over nesting colonies so they can take aerial photographs of the sites. As pelican colonies can be quite large (hundreds or thousands of nests), each photograph shows only a portion of the entire site. We have maps based on satellite images that are taken annually, which show us the shape and size of each colony site. We are enlisting your team's help to create a procedure that will allow us to estimate the number of nests in a pelican colony, based on the photograph that shows a sample of the colony, and a map that shows the size and shape of the entire site.

Attached to this memo, you will find two photographs taken at two different pelican colonies, and a map showing the size and shape of each of these colony sites. You can assume that pelicans will nest in the entire area of each site, as the habitat is uniform across a given site.

Thank you,
Alice Heart

## Colony A - Aerial Photograph



Colony A - Site Map
Area inside lines is nesting site


## Colony B - Aerial Photograph



## Colony B - Site Map

Area inside lines is nesting site


## Pelican Colonies - Part B

## Team Exercise

1. With your team, discuss and compare your responses to the individual questions. Your team should reach a consensus on which factors you will take into consideration when developing your procedure.
2. Reread the memo from Alice Heart as a group.
3. Develop a procedure for estimating the number of nests at a pelican colony, using the documents provided by the U.S. Fish and Wildlife Service and materials available in the classroom. Write a memo to Alice Heart that includes a detailed explanation of your procedure, an explanation of how you have tested your procedure, and your estimates for the number of pelican colonies in the two photographs included in your packet. Be sure to include a description of any assumptions you have made. Also include any requests for additional information or data from Alice Heart that you think might allow you to improve the accuracy of your procedure.

## OBSERVATION FORM FOR TEACHER - Pelican Colonies

Team: $\qquad$


Math Concepts Used:
What mathematical concepts and skills did the students use to solve the problem?

## Team Interactions:

How did the students interact within their team or share insights with each other?

## Data Organization \& Problem Perspective:

How did the students organize the problem data? How did the students interpret the task? What perspective did they take?

Tools:
What tools did the students use? How did they use these tools?

Miscellaneous Comments about the team functionality or the problem:

## Cycles of Assessment \& Justification:

How did the students question their problem-solving processes and their results? How did they justify their assumptions and results? What cycles did they go through?

Name $\qquad$


While the presentations are happening, choose TWO teams to evaluate. Look for things that you like about their solution and/or things that you would change in their solution. You are not evaluating their style of presenting. For example, don't write, "They should have organized their presentation better." Evaluate their solution only.

Team $\qquad$

What I liked about their solution:

What I didn't like about their solution:

Team $\qquad$

What I liked about their solution:

What I didn't like about their solution:

After seeing the other presentations, how would you change your solution? If you would not change your solution, give reasons why your solution does not need changes.

Date $\qquad$

1. What mathematical or scientific concepts and skills (e.g. ratios, proportions, forces, etc.) did you use to solve this problem?
2. How well did you understand the concepts you used?

| Not at all | A little bit | Some |
| :--- | :--- | :--- |
| Explain your choice: |  | Most of it |

3. How well did your team work together? How could you improve your teamwork?
4. Did this activity change how you think about mathematics?
