Standards for Mathematical Practice:

Standard 1: Make sense of problems and persevere in solving them

The Standard:

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Classroom Observations:

Teachers who are developing students' capacity to "make sense of problems and persevere in solving them" develop ways of framing mathematical challenges that are clear and explicit, and then check in repeatedly with students to help them clarify their thinking and their process. An early childhood teacher might ask her students to work in pairs to evaluate their approach to a problem, telling a partner to describe their process, saying "what [they] did, and what [they] might do next time." A middle childhood teacher might post a set of different approaches to a solution, asking students to identify "what this mathematician was thinking or trying out" and evaluating the success of the strategy. An early adolescence teacher might have students articulate a specific way of laying out the terrain of a problem and evaluating different starting points for solving. A teacher of adolescents and young adults might frame the task as a real-world design conundrum, inviting students to engage in a "tinkering" process of working toward mathematical proof, changing course as necessary as they develop their thinking. Visit the video excerpts at Inside Mathematics website http://www.insidemathematics.org/index.php/mathematical-practice-standards to view multiple examples of teachers engaging students in sense making and mathematical perseverance.

Students:	Because Teachers:
❖ Analyze and explain the meaning of the problem ❖ Actively engage in problem solving (Develop, carry out, and refine a plan) ❖ Show patience and positive attitudes ❖ Ask if their answers make sense ❖ Check their answers with a different method	❖ Pose rich problems and/or ask open ended questions ❖ Provide wait-time for processing/finding solutions ❖ Circulate to pose probing questions and monitor student progress ❖ Provide opportunities and time for cooperative problem solving and reciprocal teaching *Math Solutions*

Math Practice	Key Points	Students might think or do:
Make sense of problems and persevere in solving them	• explain to themselves the meaning of a problem • look for entry points to its solution • analyze givens, constraints, relationships, and goals	• "I tried that approach to solving the problem and it didn't work. What's another way I can try to solve it?" • "What's a useful way to begin working on this problem?"

Math Practice	Key Points	Students might think or do.
Make sense of problems and persevere in solving them (cont.) *MP1*	• make conjectures about the form and meaning of the solution • plan a solution pathway • consider analogous problems • monitor and evaluate progress and change course if necessary.	• They can set up a series of steps to follow to get themselves to the answer. • "There's another problem I've done that's like this that might help me here." • "This isn't working; I need to try something else." *Education Development Center, Inc.*

Make Sense of Problems and Persevere in Solving Them	Questions to Develop Mathematical Thinking
• Interpret and make meaning of the problem to find a starting point. Analyze what is given in order to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor their progress and changed their approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask themselves, "Does this make sense?" Can understand various approaches to solutions.	How would you describe the problem in your own words? How would you describe what you are trying to find? What do you notice about...? What information is given in the problem? Describe the relationship between the quantities. Describe what you have already tried. What might you change? Talk me through the steps you've used to this point. What steps in the process are you most confident about? What are some other strategies you might try? What are some other problems that are similar to this one? How might you use one of your previous problems to help you begin? How else might you organize...represent... show...? *CCSS-M Flip Books: http://katm.org/wp/common-core/*

Practice	Needs Improvement	Emerging (teacher does thinking)	Proficient (teacher mostly models)	Exemplary (students take ownership)
Make Sense of Problems and Persever e in Solving Them *Institute for Advanced Study Park City Mathematics Institute*	**Task:** ◊ Is strictly procedural. ◊ Does not require students to check solutions for errors. **Teacher:** ◊ Does not allow for wait time; asks leading questions to rush through task. ◊ Does not encourage students to individually process the tasks. ◊ Is focused solely on answers rather than processes and reasoning..	**Task:** ◊ Is overly scaffolded or procedurally "obvious". ◊ Requires students to check answers by plugging in numbers. **Teacher:** ◊ Allots too much or too little time for the task. ◊ Encourages students to individually complete the tasks, but does not ask them to evaluate the processes they used. ◊ Explains the reasons behind procedural steps. ◊ Does not check errors publicly.	**Task:** ◊ Is cognitively demanding. ◊ Has more than one entry point. ◊ Requires a balance of procedural fluency and conceptual understanding. ◊ Requires students to check solutions for errors using one other solution path. **Teacher:** ◊ Allows ample time for all students to struggle with the task. ◊ Expects students to evaluate processes implicitly. ◊ Models making sense of the task (given situation) and the proposed solution.	**Task:** ◊ Allows for multiple entry points and solution paths. ◊ Requires students to defend and justify their solution by comparing multiple solution paths.. **Teacher:** ◊ Differentiates to keep advanced students challenged during work time. ◊ Integrates time for explicit meta-cognition ◊ Expects students to make sense of task and proposed solution.

Standards for Mathematical Practice:
Standard 2: Reason abstractly and quantitatively

The Standard:

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Classroom Observations:

Teachers who are developing students' capacity to "reason abstractly and quantitatively" help their learners understand the relationships between problem scenarios and mathematical representation, as well as how the symbols represent strategies for solution. A middle childhood teacher might ask her students to reflect on what each number in a fraction represents as parts of a whole. A different middle childhood teacher might ask his students to discuss different sample operational strategies for a patterning problem, evaluating which is the most efficient and accurate means of finding a solution. Visit the video excerpts below to view these teachers engaging their students in abstract and quantitative reasoning.

http://www.insidemathematics.org/index.php/mathematical-practice-standards to view multiple examples of teachers engaging students in reasoning abstractly and quantitatively.

Students:	Because Teachers:
❖ Represent a problem with symbols ❖ Explain their thinking ❖ Use numbers flexibly by applying properties of operations and place value ❖ Examine the reasonableness of their answers/calculations	❖ Ask students to explain their thinking regardless of accuracy ❖ Highlight flexible use of numbers ❖ Facilitate discussion through guided questions and representations ❖ Accept varied solutions/representations *Math Solutions*

Math Practice	Key Points	Students might think or do:
Reason Abstractly and Quantitatively	• make sense of quantities and their relationships in problem situations • *decontextualize* - abstract and represent a problem situation symbolically and manipulate those symbols without attending to their referents • *contextualize* - pause during problem solving to connect symbolic work back to the context of the problem	• "How can I capture important information in a diagram or model?" • "What solution path does this diagram or model imply?" • "OK, I've done all these calculations; now, what does that mean in the problem? Does my answer make sense for answering this problem?"

Reason Abstractly and Quantitatively (cont.) *MP2*	• Pay attention to the important quantities and relationships between them • use representations to highlight those relationships and the underlying mathematical structure of a problem	• Given the problem: *There are 3/5 as many boys as girls. If there are 45 boys, how many girls are there?,* a student can create a diagram that shows the relationship between the number. *Education Development Center, Inc.*

Reason Abstractly and Quantitatively	Questions to Develop Mathematical Thinking
• Make sense of quantities and their relationships. • Are able to decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	What do the numbers used in the problem represent? What is the relationship of the quantities? How is____related to____? What properties might we use to find a solution? What does____mean to you? (e.g., symbol, quantity, diagram) How did you decide in this task that you needed to use . . . ? Could we have used another operation or property to solve this task? Why or why not? *CCSS-M Flip Books: http://katm.org/wp/common-core/*

Practice	Needs Improvement	Emerging (teacher does thinking)	Proficient (teacher mostly models)	Exemplary (students take ownership)
Reason Abstractly and Quantitatively	**Task:** ◊ Lacks context. ◊ Does not make use of multiple representations or solution paths. **Teacher:** ◊ Does not expect students to interpret representations. ◊ Expects students to memorize procedures with no connection to meaning.	**Task:** ◊ Is embedded in a contrived context. **Teacher:** ◊ Expects students to model and interpret tasks using a single representation. ◊ Explains connections between procedures and meaning.	**Task:** ◊ Has realistic context. ◊ Requires students to frame solutions in a context. ◊ Has solutions that can be expressed with multiple representations. **Teacher:** ◊ Expects students to interpret and model using multiple representations. ◊ Provides structure for students to connect algebraic procedures to contextual meaning. ◊ Links mathematical solution with a question's answer.	**Task:** ◊ Has relevant realistic context. **Teacher:** ◊ Expects students to interpret, model, and connect multiple representations. ◊ Prompts students to articulate connections between algebraic procedures and contextual meaning. *Institute for Advanced Study* *Park City Mathematics Institute*

Standards for Mathematical Practice:

Standard 3: Construct Viable Arguments and Critique the Reasoning of Others

The Standard:

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Classroom Observations:

Teachers who are developing students' capacity to "construct viable arguments and critique the reasoning of others" require their students to engage in active mathematical discourse. This might involve having students explain and discuss their thinking processes aloud, or signaling agreement/disagreement with a hand signal. A middle childhood teacher might post multiple approaches to a problem and ask students to identify plausible rationales for each approach as well as any mistakes made by the mathematician. An early adolescence teacher might post a chart showing a cost-analysis comparison of multiple DVD rental plans and ask his students to formulate and defend a way of showing when each plan becomes most economical. A teacher of adolescents and young adults might actively engage her students in extended conjecture about conditions for proof in the construction of quadrilaterals, testing their assumptions and questioning their approaches. Visit the video excerpts at Inside Mathematics website: http://www.insidemathematics.org/index.php/mathematical-practice-standards to view multiple examples of teachers engaging students in formulating, critiquing and defending arguments of mathematical reasoning.

Students:	Because Teachers:
❖ Make reasonable guesses to explore their ideas ❖ Justify solutions and approaches ❖ Listen to the reasoning of others, compare arguments, and decide if the arguments of others makes sense ❖ Ask clarifying and probing questions	❖ Provide opportunities for students to listen to or read the conclusions and arguments of others ❖ Establish and facilitate a safe environment for discussion ❖ Ask clarifying and probing questions ❖ Avoid giving too much assistance (e.g., providing answers or procedures) *Math Solutions*

Math Practice	Key Points	Students might think or do:
Construct viable arguments and critique the reasoning of others *MP 3*	• make conjectures and build a logical progression of statements to explore the truth of their conjectures • analyze situations by breaking them into cases • recognize and use counterexamples • justify conclusions, communicate them to others, and respond to the arguments of others • distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is	• A student can state a rule for a pattern, and can explain why their rule works for that pattern. • When someone claims "multiplying two numbers gives you an answer bigger than either the numbers," a student can think about: - what happens when you multiply 2 whole numbers; - what happens when you multiply by a fraction; - what happens when you multiply 2 fractions *Education Development Center, Inc.*

Construct viable arguments and critique the reasoning of others.	Questions to Develop Mathematical Thinking
• Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	What mathematical evidence would support your solution? How can we be sure that...? How could you prove that...? Will it still work if...? What were you considering when...? How did you decide to try that strategy? How did you test whether your approach worked? How did you decide what the problem was asking you to find? (What was unknown?) Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? What is the same and what is different about...? How could you demonstrate a counter-example? *CCSS-M Flip Books: http://katm.org/wp/common-core/*

Practice	Needs Improvement	Emerging (teacher does thinking)	Proficient (teacher mostly models)	Exemplary (students take ownership)
Construct Viable Arguments and Critique Reasoning of Others	**Task:** ◊ Is ambiguously stated. **Teacher:** ◊ Does not ask students to present arguments or solutions. ◊ Expects students to follow a given solution path without opportunities to make conjectures.	**Task:** ◊ Is not at the appropriate level. **Teacher:** ◊ Does not help students differentiate between assumptions and logical conjectures. ◊ Asks students to present arguments but not to evaluate them. ◊ Allows students to make conjectures without justification.	**Task:** ◊ Avoids single steps or routine algorithms. **Teacher:** ◊ Identifies students' assumptions. ◊ Models evaluation of student arguments. ◊ Asks students to explain their conjectures.	**Teacher:** ◊ Helps students differentiate between assumptions and logical conjectures. ◊ Prompts students to evaluate peer arguments. ◊ Expects students to formally justify the validity of their conjectures. *Institute for Advanced Study* *Park City Mathematics Institute*

Standards for Mathematical Practice:

Standard 4:Model With Mathematics

The Standard:

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Classroom Observations:

Teachers who are developing students' capacity to "model with mathematics" move explicitly between real-world scenarios and mathematical representations of those scenarios. A middle childhood teacher might pose a scenario of candy boxes containing multiple flavors to help students identify proportions and ratios of flavors and ingredients. An early adolescence teacher might represent a comparison of different DVD rental plans using a table, asking the students whether or not the table helps directly compare the plans or whether elements of the comparison are omitted. A teacher of adolescents and young adults might pose a "kite factory" scenario, in which advanced students are asked to determine the conditions for always creating a particular shape of kite given the dimensions of the diagonals and the angle of intersection. Visit the video excerpts at Inside Mathematics website: http://www.insidemathematics.org/index.php/mathematical-practice-standards to view multiple examples of teachers engaging students in mathematical modeling.

Students:	Because Teachers:
❖ Apply prior knowledge to new problems and reflect ❖ Use representations to solve real life problems ❖ Apply formulas and equations where appropriate	❖ Pose problems connected to previous concepts ❖ Provide a variety of real world contexts ❖ Use intentional representations *Math Solutions*

Math Practice	Key Points	Students might think or do:
Model With Mathematics *MP 4*	• make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later • identify important quantities in a practical situation • map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas • analyze those relationships mathematically to draw conclusions • interpret their mathematical results in the context of the situation	• A student can state a rule for a pattern, and can explain why their rule works for that pattern. • When someone claims "multiplying two numbers gives you an answer bigger than either the numbers," a student can think about: - what happens when you multiply 2 whole numbers; - what happens when you multiply by a fraction; - what happens when you multiply 2 fractions *Education Development Center, Inc.*

Model With Mathematics	Questions to Develop Mathematical Thinking
• Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the math they know to solve problems in everyday life. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving or revising the model. • Ask themselves, "How can I represent this mathematically?"	What number model could you construct to represent the problem? What are some ways to represent the quantities? What's an equation or expression that matches the diagram..., number line.., chart..., table..? Where did you see one of the quantities in the task in your equation or expression? Would it help to create a diagram, graph, table...? What are some ways to visually represent...? What formula might apply in this situation? *CCSS-M Flip Books: http://katm.org/wp/common-core/*

Practice	Needs Improvement	Emerging (teacher does thinking)	Proficient (teacher mostly models)	Exemplary (students take ownership)
Model With Mathematics	**Task:** ◊ Requires students to identify variables and to perform necessary computations. **Teacher:** ◊ Identifies appropriate variables and procedures for students. ◊ Does not discuss appropriateness of models.	**Task:** ◊ Requires students to identify variables and to compute and interpret results. **Teacher:** ◊ Verifies that students have identified appropriate variables and procedures. ◊ Explains the appropriateness of model.	**Task:** ◊ Requires students to identify variables, compute results, and report findings using a mixture of representations. ◊ Illustrates the relevance of the mathematics involved. ◊ Requires students to identify extraneous or missing information. **Teacher:** ◊ Asks questions to help students identify appropriate variables and procedures. ◊ Facilitates discussions in evaluating the appropriateness of model.	**Task:** ◊ Requires students to identify variables, compute and interpret results, and justify the reasonableness of their results and procedures. **Teacher:** ◊ Expects students to justify their choice of variables and procedures. ◊ Gives students opportunity to evaluate the appropriateness of model. *Institute for Advanced Study* *Park City Mathematics Institute*

Standards for Mathematical Practice:
Standard 5. Use Appropriate Tools Strategically

The Standard:

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Classroom Observations:

Teachers who are developing students' capacity to "use appropriate tools strategically" make clear to students why the use of manipulatives, rulers, compasses, protractors, and other tools will aid their problem solving processes. A middle childhood teacher might have his students select different color tiles to show repetition in a patterning task. A teacher of adolescents and young adults might have established norms for accessing tools during the students' group "tinkering processes," allowing students to use paper strips, brass fasteners, and protractors to create and test quadrilateral "kite" models. Visit the video excerpts at Inside Mathematics website: http://www.insidemathematics.org/index.php/mathematical-practice-standards to view multiple examples of teaching students to use appropriate tools strategically.

Students:	Because Teachers:
❖ Select and use tools strategically (and flexibly) to visualize, explore, and compare information ❖ Use technological tools and resources to solve problems and deepen understanding	❖ Make appropriate tools available for learning (calculators, concrete models, digital resources, pencil/paper, compass, protractor, etc.) ❖ Use tools with their instruction *Math Solutions*

Math Practice	Key Points	Students might think or do:
Use Appropriate Tools Strategically (Ex: pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software.)	• Are familiar with tools appropriate for their grade or course and can make sound decisions about when each of these tools might be helpful. • Identify relevant external mathematical resources, such as digital content located on a website, and to use them to pose or solve problems.	• A student wants to see how the difference between values in a table changes, so she begins by making a table, then decides to put the information in a spreadsheet to more easily do the calculations, and draw conclusions from the results. • A student is having trouble visualizing a situation with a geometric shape, so he creates it in a geometry software application and is able to move the shape around to see how some parts of the shape change while keeping certain characteristics of the shape the same. *Education Development Center, Inc.*

Use Appropriate Tools Strategically *MP5*	Questions to Develop Mathematical Thinking
• Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	What mathematical tools could we use to visualize and represent the situation? What information do you have? What do you know that is not stated in the problem? What approach are you considering trying first? What estimate did you make for the solution? In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...? What can using a______show us, that______may not? In what situations might it be more informative or helpful to use...? *CCSS-M Flip Books: http://katm.org/wp/common-core/*

Practice	Needs Improvement	Emerging (teacher does thinking)	Proficient (teacher mostly models)	Exemplary (students take ownership)
Use Appropriate Tools Strategically	**Task:** ◊ Does not incorporate additional learning tools. **Teacher:** ◊ Does not incorporate additional learning tools.	**Task:** ◊ Lends itself to one learning tools. ◊ Does not involve mental computations or estimation. **Teacher:** ◊ Demonstrates use of appropriate learning tool.	**Task:** ◊ Lends itself to multiple learning tools. ◊ Gives students opportunity to develop fluency in mental computations. **Teacher:** ◊ Choose appropriate learning tools for student use. ◊ Models error checking by estimation.	**Task:** ◊ Requires multiple learning tools (i.e., graph paper, calculator, manipulatives) ◊ Requires students to demonstrate fluency in mental computations. **Teacher:** ◊ Allows students to choose learning tools. ◊ Creatively finds appropriate alternatives where tools are not available. *Institute for Advanced Study* *Park City Mathematics Institute*

Standards for Mathematical Practice:
Standard 6. Attend to Precision

The Standard:

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Classroom Observations:

Teachers who are developing students' capacity to "attend to precision" focus on clarity and accuracy of process and outcome in problem solving. A middle childhood teacher might engage his students in a "number talk" in which students use an in/out table and a plotted graph to "guess [the teacher's] number." An early adolescence teacher might distribute cards with different symbol strings to his students, asking them to mingle to group and categorize their symbol strings, explaining and defending their groupings. A teacher of adolescents and young adults might continually probe her students to defend whether their requirements for a particular quadrilateral will always be the case, or whether there are some flaws in their group's thinking that they need to refine and correct. Visit the video excerpts at Inside Mathematics website: http://www.insidemathematics.org/index.php/mathematical-practice-standards to view multiple examples of teachers engaging students in attending to precision.

Students:	Because Teachers:
❖ Calculate accurately and efficiently ❖ Explain their thinking using mathematics vocabulary ❖ Use appropriate symbols and specify units of measure	❖ Recognize and model efficient strategies for computation ❖ Use (and challenge students to use) mathematics vocabulary precisely and consistently *Math Solutions*

Math Practice	Key Points	Students might think or do:
Attend to Precision	• Use clear definitions in discussion with others and in their own reasoning. • State the meaning of the symbols they choose, including using the equal sign consistently and appropriately. • Are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. • Express numerical answers with a degree of precision appropriate for the problem context.	• A student rewrites his explanation to a problem using appropriate mathematics vocabulary. • A student learns why it is incorrect to write $14 + 4 = 18 + 5 = 23 \times 2 = 46$ • "My calculator says 3.581279, but since I'm asked to find the number of inches, that's not a number that makes sense to write for a measurement in inches. I'll say 3.5" or 3.6". *Education Development Center, Inc.*

Attend to Precision *MP6*	**Questions to Develop Mathematical Thinking**
• Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	What mathematical terms apply in this situation? How did you know your solution was reasonable? Explain how you might show that your solution answers the problem. Is there a more efficient strategy? How are you showing the meaning of the quantities? What symbols or mathematical notations are important in this problem? What mathematical language...,definitions..., properties can you use to explain How could you test your solution to see if it answers the problem? *CCSS-M Flip Books: http://katm.org/wp/common-core/*

Practice	**Needs Improvement**	**Emerging** (teacher does thinking)	**Proficient** (teacher mostly models)	**Exemplary** (students take ownership)
Attend to Precision	**Task:** ◊ Gives imprecise instructions. **Teacher:** ◊ Does not intervene when students are being imprecise ◊ Does not point out when students fail to address the question completely or directly.	**Task:** ◊ Has overly detailed or wordy instructions. **Teacher:** ◊ Inconsistently intervenes when students are imprecise. ◊ Identifies incomplete responses but does not require student to formulate further response.	**Task:** ◊ Has precise instructions. **Teacher:** ◊ Consistently demands precision in communication and in mathematical solutions. ◊ Identifies incomplete responses and asks student to revise their response.	**Task:** ◊ Includes assessment criteria for communication of ideas. **Teacher:** ◊ Demands and models precision in communication and in mathematical solutions. ◊ Encourages student to identify when others are not addressing the question completely. *Institute for Advanced Study* *Park City Mathematics Institute*

Standards for Mathematical Practice:
Standard 7. Look For and Make Use of Structure

The Standard:

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Classroom Observations:

Teachers who are developing students' capacity to "look for and make use of structure" help learners identify and evaluate efficient strategies for solution. An early childhood teacher might help students identify why using "counting on" is preferable to counting each addend by one, or why multiplication or division can be preferable to repeated addition or subtraction. A middle childhood teacher might help his students discern patterns in a function table to "guess my rule." A teacher of adolescents and young adults might focus on exploring geometric processes through patterns and proof. Visit the video excerpts at Inside Mathematics website: http://www.insidemathematics.org/index.php/mathematical-practice-standards to view multiple examples of teachers engaging students in identifying and making use of mathematical structure.

Students:	Because Teachers:
❖ Look for, develop, and generalize relationships and patterns ❖ Apply reasonable thoughts about patterns and properties to new situations	❖ Provide time for applying and discussing properties ❖ Ask questions about the application of patterns ❖ Highlight different approaches for solving problems *Math Solutions*

Math Practice	Key Points	Students might think or do:
Look For and Make Use of Structure	•Look for similar mathematical structures across Seemingly different problems •Use those similarities to help them reason about how to solve a problem •Can step back for an overview and shift perspective •Can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.	• "Since (problem A) and (problem B) are structurally the same, what do I know about solving (prob A) that will me think about solving (prob B)?" • "Figuring out what to do with 3(x + 2) is just like the work I did in (?th) grade when I learned that 7 x 8 is the same as (7 x 3) + (7 x 5)." • Would recognize that 17 + 2 (x + 1) will be odd because 2 (x + 1) is even (since it's 2 times some number) and 17 is odd, and an odd amount + an even amount will be odd. • Noticing that all numbers that have a remainder of 4 when divided by 5 will end in either 4 or 9. • "I recognize that 1/3 (A + B + C) is really just the same as finding the average of 3 numbers." *Education Development Center, Inc.*

Look For and Make Use of Structure *MP7*	Questions to Develop Mathematical Thinking
• Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	What observations do you make about...? What do you notice when...? What parts of the problem might you eliminate...,simplify...? What patterns do you find in...? How do you know if something is a pattern? What ideas that we have learned before were useful in solving this problem? What are some other problems that are similar to this one? How does this relate to...? In what ways does this problem connect to other mathematical concepts? *CCSS-M Flip Books: http://katm.org/wp/common-core/*

Practice	Needs Improvement	Emerging (teacher does thinking)	Proficient (teacher mostly models)	Exemplary (students take ownership)
Look For and Make Use of Structure	**Task:** ◊ Requires students to automatically apply an algorithm to a task without evaluating its appropriateness. **Teacher:** ◊ Does not recognize students for developing efficient approaches to the task. ◊ Requires students to apply the same algorithm to a task although there may be other approaches.	**Task:** ◊ Requires students to analyze a task before automatically applying an algorithm. **Teacher:** ◊ Identifies individual students' efficient approaches, but does not expand understanding to the rest of the class. ◊ Demonstrates the same algorithm to all related tasks although there may be other more effective approaches.	**Task:** ◊ Requires students to analyze a task and identify more than one approach to the problem. **Teacher:** ◊ Facilitates all students in developing reasonable and efficient ways to accurately perform basic operations. ◊ Continuously questions students about the reasonableness of their intermediate results.	**Task:** ◊ Requires students to identify the most efficient solution to the task. **Teacher:** ◊ Prompts students to identify mathematical structure of the task in order to identify the most effective solution path. ◊ Encourages student to justify their choice of algorithm or solution path. *Institute for Advanced Study* *Park City Mathematics Institute*

Standards for Mathematical Practice:
Standard 8. Look For and Express Regularity in Repeated Reasoning

The Standard:

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x2 + x + 1), and (x – 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Classroom Observations:

Integrating Standard Eight into classroom practice is not only a matter of planning for lessons that occasion students to look for general methods and shortcuts. It also requires teachers to attend to and listen closely to their students' noticings and "a-ha moments," and to follow those a-ha moments so that they generalize to the classroom as a whole. The video clips at Inside Mathematics website: http://www.insidemathematics.org/index.php/mathematical-practice-standards are intended to show as a composite how teachers create the conditions for students to look for and express regularity in repeated reasoning, and follow and elaborate students' thinking when they begin to make these connections.

Students:	Because Teachers:
❖ Look for methods and shortcuts in patterns and repeated calculations ❖ Evaluate the reasonableness of results and solutions	❖ Provide tasks and problems with patterns ❖ Ask about answers before and reasonableness after computations *Math Solutions*

Math Practice	Key Points	Students might think or do:
Look For and Express Regularity in Repeated Reasoning	•Look for general methods and for shortcuts in the calculations, understanding why the shortcut works. •Maintain oversight of the problem-solving process, while attending to the details of the calculations •Continually evaluate the reasonableness of their intermediate results.	• "When I divide 15 by 9, the 9 keeps 'going in' 6 times…over and over again. That means I have a repeating decimal." • "I solve this problem using 8 adults. Then I solved it using 10 adults, 12 adults, and 20 adults. What's the same about my solution steps each time? How can that help me describe a process or an equation for the problem?" • (When a student is immersed in some calculations, they can stop and think) "Wait, where am I going with this? What does 5.76 represent? Where am I in the process of solving this problem?" • "Wait, I can't just write 7.2 because you can't have 7.2 children in a group." *Education Development Center, Inc.*

Look For and Express Regularity in Repeated Reasoning *MP8*	Questions to Develop Mathematical Thinking
• See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results.	Will the same strategy work in other situations? Is this always true, sometimes true or never true? How would we prove that...? What do you notice about...? What is happening in this situation? What would happen if...? Is there a mathematical rule for...? What predictions or generalizations can this pattern support? What mathematical consistencies do you notice ? *CCSS-M Flip Books: http://katm.org/wp/comm*

Practice	Needs Improvement	Emerging (teacher does thinking)	Proficient (teacher mostly models)	Exemplary (students take ownership)
Look For and Express Regularity in Repeated Reasoning	**Task:** ◊ Is disconnected from prior and future concepts. ◊ Has no logical progression that leads to pattern recognition. **Teacher:** ◊ Does not show evidence of understanding the hierarchy within concepts. ◊ Presents or examines task in isolation.	**Task:** ◊ Is overly repetitive or has gaps that do not allow for development of a pattern. **Teacher:** ◊ Hides or does not draw connections to prior or future concepts.	**Task:** ◊ Reviews prior knowledge and requires cumulative understanding. ◊ Lends itself to developing a pattern or structure. **Teacher:** ◊ Connects concept to prior and future concepts to help students develop an understanding of procedural shortcuts. ◊ Demonstrates connections between tasks.	**Task:** ◊ Addresses and connects to prior knowledge in a non-routine way. ◊ Requires recognition of pattern or structure to be completed. **Teacher:** ◊ Encourages students to connect task to prior concepts and tasks. ◊ Prompts students to generate exploratory questions based on current task. ◊ Encourages students to monitor each other's intermediate results. *Institute for Advanced Study* *Park City Mathematics Institute*